Using a Multi-Representational Design Framework to Develop and Evaluate a Dynamic Simulation Environment

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Abstract: Many researchers have begun to call for environments to be developed which specifically recognize the distinctive contribution that multiple external representations (MERs) can bring to learning. In this paper, we present the conceptual framework that forms the basis for our approach to understanding learning with MERs and describe how we have embedded in the design of an instructional simulation that uses dynamic visualizations. We discuss briefly the results of an initial evaluation of the system and its implications for testing the framework.

Introduction

Learning with multiple (external) representations (MERs) has been recognized as potentially powerful way of facilitating understanding for many years. Early research concentrated on the ways that presenting pictures alongside text could improve readers' memory for or comprehension of text (e.g. Mayer 1993; Winn, 1987). In the last two decades years, the debate has widened to include an extensive variety of representational formats including animations, sound, video an dynamic simulations. Much research has focussed on whether particular types of representations bring associated benefits for learning and problem solving. For example, Larkin & Simon (1987) proposed that diagrams exploit perceptual processes by grouping together relevant information that make processes such as search and recognition easier. Alternatively, tables tend to make information explicit, emphasise empty cells that directs attention to unexplored alternatives, and allow quicker and more accurate readoff. Descriptive representations are symbolic in nature whereas depictive representations are iconic. Thus, depictive representations are most useful to provide concrete information about specific instantiations of the object of interest and descriptive representations to express general information (Schnotz, 2002)

Other research has focussed on the unique potential and problems of learning simultaneously with more than one representation (e.g. Van Someren, Reimann, Bozhimen, & de Jong, 1998). A number of studies have found that learners benefit from either constructing or being presented with MERs (e.g. Cox & Brna, 1995, Mayer & Sims, 1994). These studies are supported by a number of theoretical positions about the benefits of MERs. For example, Dienes (1973) argues that perceptual variability (the same concepts represented in varying ways) provides learners with the opportunity to build abstractions about mathematical concepts. In cognitive flexibility theory (Spiro & Jehng, 1990) the ability to construct and switch between multiple perspectives of a domain is fundamental to successful learning. Unfortunately, there are many studies that have shown that learners can fail to benefit from these proposed advantages of MERs (e.g. Yerushalmy, 1991).

It is apparent that MERS are used for diverse pedagogical goals, include numerous forms of representational system and are designed in many different ways. Therefore, generalisations concerning the effectiveness of multi-representational learning are likely to be difficult to achieve. However, to derive general principles for effective learning with MERs we need to be able to predict the conditions under which more than one representation will be beneficial. The DeFT (Design, Functions, Tasks) framework aims to do so by describing different pedagogical functions that multiple representations can play, the design parameters that are unique to learning with more than one representation and the cognitive tasks that must be undertaken by a learner when interacting with MERs. In the remainder of this paper, each subcomponent of DeFT will be introduced. We will then show how we have used it to design an instructional simulation that utilizes multiple dynamic representations and discuss its evaluation.

DeFT

The DeFT Framework (e.g. Ainsworth & Van Labeke, 2001) provides an account of the different pedagogical *functions* that MERs can play, the *design* parameters that are unique to learning with more than one representation and the cognitive *tasks* that must be undertaken by a learner when interacting with MERs.

Functions

There are three key functions of MERs: to complement, constrain and construct (see Figure 1 and Ainsworth, 1999).

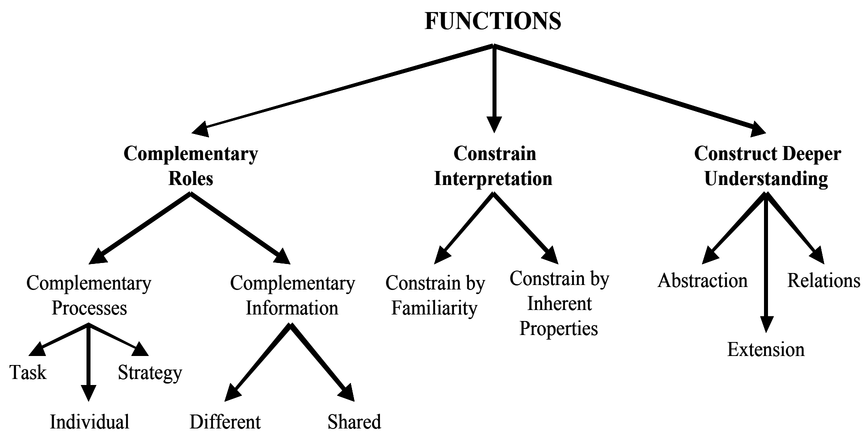

Figure 1. A Functional Taxonomy of Multiple Representations

Complementary Functions

When MERs complement each other they do so because they differ either in the information each expresses or in the processes each supports. By combining representations that differ in these ways, it is hoped that learners will benefit from the advantages of each of the individual representations.

Multiple representations tend to be used to provide complementary information when a single representation would be insufficient to carry all the information about the domain or would be too complicated for learners to interpret if it did so.

When MERs are used to exploit the varying computational processes supported by different representations they provide learners with the opportunities to draw alternative inferences about the domain (e.g. Larkin & Simon, 1987, Cox & Brna, 1995; Stenning & Oberlander, 1995). This is one of the most common reasons to use MERs and can be advantageous for a number of reasons.

If learners are presented with a choice of representations they can select the representations that best suits their needs. There is some evidence that this can improve learning (Plass, 1998). They may do this due to differing expertise with alternative forms of representation or because of more stable individual differences (e.g. Winn, 1987), although there is not necessarily a face-valid relationship (Roberts, 1997).

The "match-mismatch" conjecture by Gilmore & Green (1984) proposes that performance is most likely to be facilitated when the structure of information required by the problem matches the form provided by the representational notation. Thus by providing multiple representations learners can benefit from choosing the best representation for the current task..

Multiple representations can encourage learners to try more than one strategy to solve a problem. Tabachneck, Koedinger & Nathan (1994) examined the representations that learners created to solve algebra word problems and found each representation was associated with different strategies. The use of MERs and hence multiple strategies was about twice as effective as any strategy used alone using. As each strategy had inherent weaknesses, switching between strategies made problem solving more successful by compensating for this.

Constraining Functions

A second use of multiple representations is to help learners develop a better understanding of a domain by using one representation to constrain their interpretation of a second representation. This can be achieved in two ways: by employing a familiar representation to support the interpretation of a less familiar one. For example, familiar concrete representations such as simple animations are often employed in simulations to support interpretation of complex and unfamiliar representations such as graphs. Graphs can be used to constrain interpretation of equations. Yerushalmy (1989) describes a multi-representational learning environment for teaching algebraic transformations. It presents users with an algebraic window where they transform algebraic expressions. It also provides three graphs: the first displays a graph of the original expression; the second displays the current transformed expressions and; the third describes any difference between the two expressions. Consequently, learners are encouraged to check that their transformations are correct as graphs should not change if a transformation was legal.

Sometimes these constraints are achieved by taking advantages of inherent properties of representations. Graphical representations are generally more specific than sentential representations (*e.g.* Stenning & Oberlander, 1995). If someone is provided with a representation in a natural language expression such as 'the knife is beside the fork', there is inherent ambiguity about which side of the knife the fork has been placed. This is not possible when representing the same world pictorially, since the fork must be shown as either to the left or to the right of the knife (*e.g.* Ehrlich & Johnson-Laird, 1982). So, when these two representations are presented together, interpretation of the first (ambiguous) representation may be constrained by the second (specific) representation.

Constructing Functions

Thirdly, MERs can support the construction of deeper understanding when learners integrate information from MERS to achieve insight it would be difficult to achieve with only a single representation. 'Deeper understanding' is considered as abstraction, extension or relational understanding. Abstraction is the process by which learners create mental entities that serve as the basis for new procedures and concepts at a *higher* level of organization. For example, Dienes (1973) argues that perceptual variability (the same concepts represented in varying ways) provides learners with the opportunities for building such abstractions. Extension can be considered as a way of extending knowledge that a learner has from a known to an unknown to representation, but without fundamentally reorganizing the nature of that knowledge. Finally, we consider relational understanding to be the process by which two known representations are associated again without reorganization of knowledge. The goal of teaching relation between representations can sometimes be an end in itself. For example, much emphasis is placed on learning how to construct a graph given an equation (*e.g.* Dugdale, 1982). . It should also be noted that whether representations serve to support abstraction, extension or relations often depend upon learners' knowledge not system designers intent. For example, one learner may be familiar with tables and extent their knowledge to graphs (extension), another may already be familiar with both but not consider their relationship (relation). The differences between these functions of MERs are quite subtle and all may be present at some stage in the life cycle of encouraging deeper understanding with a multi-representational environment.

Tasks

Learners are faced with complex learning tasks when they are first presented with a novel multi-representational system. The first learning task facing any user of a representation is to understand each representation. They must know how a representation encodes and presents information (the 'format'). In the case of a graph, the format would be attributes such as lines, labels, and axes. They must also learn what the 'operators' are for a given representation. For a graph, operators to be learnt include how to find the gradients of lines, maxima and minima, intercepts, *etc*

Secondly, interpretation of representations is an inherently contextualised activity (e.g., Shah & Hoeffner, 2002) as learners must also come to understand the relation between the representation and the domain that it represents. This task will be particularly difficult for learning with MERs as opposed to problem solving or professional practise, as learners are attempting to forge this understanding based upon incomplete domain knowledge. Learners need to determine which operators to apply to the representation to retrieve the relevant domain information.

In some circumstances, learners are faced with choosing the representations they are going to use. Consequently, they face the additional task of determining which are the most appropriate representations for a particular task. This may require them to consider both the aspects of the domain and their representational preferences, which may be influence by both prior experiences and individual differences.

Learners are faced with further cognitive tasks if constructing representations rather than interpreting presented representations. This can sometimes add a source of error if learners construct their representations inaccurately.

However, it should make the process of interpretation easier as learners are familiar with the format and operators of representations and how to relate them to the domain, (Cox, 1996). Cox also found that learners could draw the correct inference from incorrect representations.

Finally, the task that is unique to learning with MERs is learners should understand the relation between representations. Unfortunately, a very large number of studies have observed that learners find translating between representations difficult (e.g. Ainsworth Bibby & Wood, 2002, Anzai, 1991; Schoenfeld, Smith & Arcavi, 1993). This contrasts with expert performance where it is apparent that a characteristic of expertise is many domains is the ability to integrate different representational formats (e.g. Kozma, Chin, Russel & Marx, 2000; Tabachneck, et al, 1997).

Design

There are a number of factors that influence the way that learners process MERs. DeFT considers these as design dimensions that uniquely apply to multi-representational systems. They are proposed to be: (a) the way that information is distributed between MERs; (b) the form of the representational system; (c) translation between representations; (d) the number of representations employed and; (e) the sequencing of representations

Information Multi-representational systems allow for flexibility in the way that information is distributed between the representations and, consequently, the redundancy of information between representations. At one extreme, each representation in the multi-representational system could express the same information. Here, the only difference between the representations is in their computational properties. At the other extreme, each representation could convey completely different information. Multi-representational systems can also be partially redundant, so that some of the information is constant across (some of) the representations.

Form: A multi-representational system can contain representations of different computational properties. Much research has focussed heterogeneous systems, i.e. ones that combine text and graphics. In theories such of those of Mayer (1997) and Schnotz (2002) particular benefits occur from the simultaneous presentation of text and pictures. Unfortunately, Ainsworth, et al (2002) found that children using heterogeneous MERs to learn estimation did not improve their performance whereas those using homogenous representations did.

Translation: The degree of support provided for mapping between two representations. This can range from no support through to highlighting and on to full dyna-linking where behaviour on one representation is reflected onto another. Recently, given recognition of the difficulties learners face in integrating information from MERs a number of alternative approaches for translation have been proposed. Some researchers recommend dyna-linking (e.g. Kaput, 1992). Ploetzner, Bodemer, & Feuerlein, I (2002) proposes an approached based on structure mapping where learners are encouraged to systematically map familiar aspects of a representation onto an unfamiliar representation. Van-Labeke & Ainsworth (2001) suggested an approach based on scaffolding theory known as contingent translation, which fades the degree of system support as the learner experiences grows (see below for more details).

Number: The most visible decision about the design of a multi-representational learning environment is how many representations to employ. By definition, a multi-representational environment uses at least two representations, but many systems use more than that. A related issue is how many representations to use simultaneously?

Sequence: Many systems present only a subset of their representations at one time; consequently further decisions must be made. The first issue is the order in which the representations should be presented. When a sequence has been determined, then further complexity is provided by the necessity of deciding at what point to add a new representation or switch between the representations. Additionally, sometimes these decisions are under system control and sometimes learners can make some or all of these decisions for themselves.

Using DeFT in System Design

DEMIST (Van Labeke & Ainsworth, 2001) aims to support learner in the development of their knowledge of the concepts and representations important in understanding population dynamics. It provides a number of mathematical models, for example, the Lotka-Volterra model of predation for learners to explore. To investigate these models, users are presented with a potentially very large set of representations. Hence, DEMIST (see Figure 2) also aims to support learners understanding of how domain general representations such as X-Time graphs are used in this domain, to introduce them to the specific representations of population dynamics (such as phaseplots and life tables) and to encourage their understanding of the relationship between these representations.

DEMIST was designed to provide an experimental toolkit to explore the DeFT framework. Consequently, it does not try to embody any particular function of MERs; instead it aims to be sufficiently flexible that an author can create environments where all functions could be achieved. Thus, it is possible to use a concrete animation to help support understanding of an unfamiliar phaseplot or use tables, graphs and equations to support different computational properties. Pairs of representations can range from fully redundant to non-redundant. It also allows systematic manipulations of DeFT's design parameters. Thus, authors have control over factors such as the nature of the representations, the number allowed to presented simultaneously, whether to present representations in a sequence or under learner control and the nature of translation between representations.

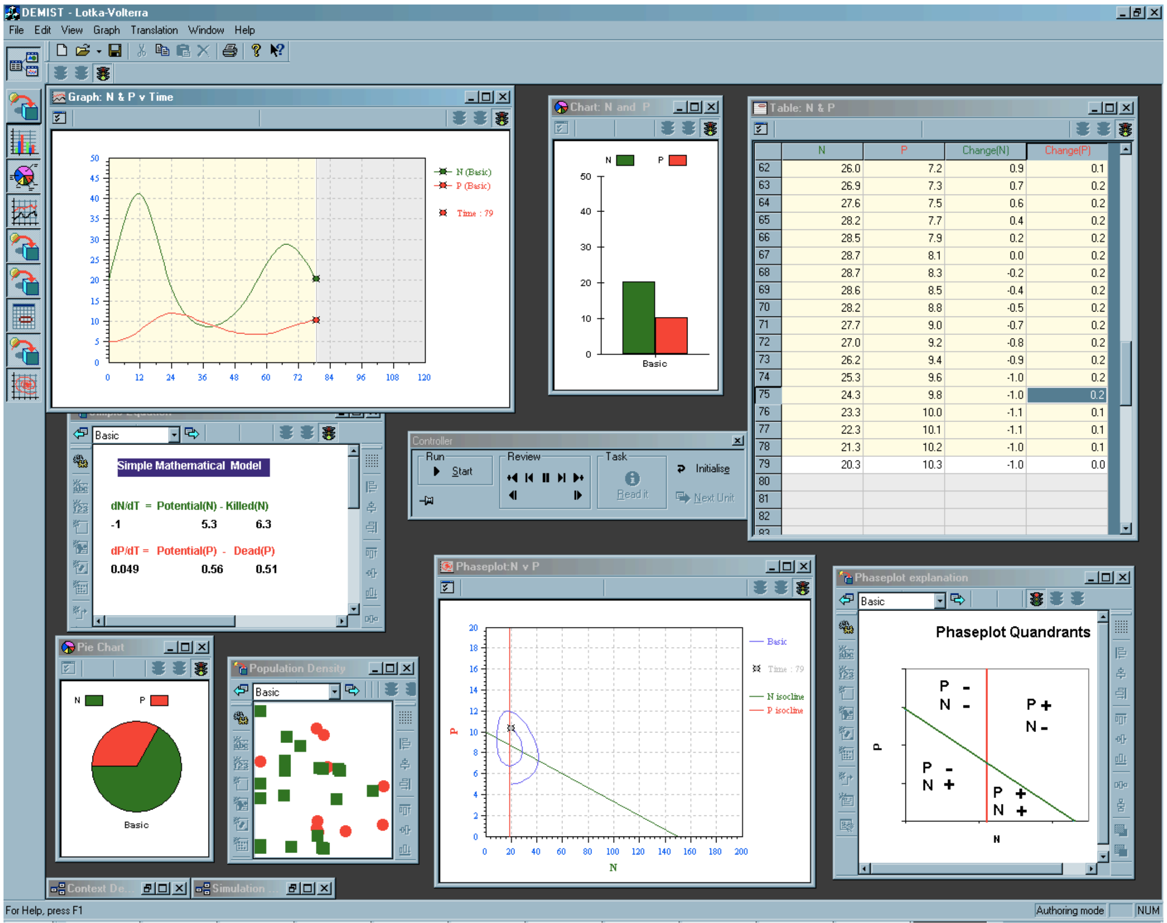

Figure 2. DEMIST displaying 8 representations for the Lotka-Volterra Model

DEMIST is build around instructional scenarios. The basis for its design is a formal description of an instructional simulation that describes the task of authoring simulations with SIMQUEST (Kuypers, 1998). Each scenario consists in a sequence of *Learning Units* that instantiate a particular mathematical model. The parameters of the mathematical model are combined as *experimental sets* that can be instantiated by various sets of initial conditions. This allows the learner to explore the same model under different experimental conditions. Each of these Learning Units includes a set of representations such as tables, XY-Graphs, Histograms, Dynamic Animations, etc. They can display one or many of the variables and parameters extracted from the mathematical model. Representations can be automatically displayed or only shown when the learner requests then and the order in which they appear can be specified by an author or left under learner control. To allow implementation of contingent translation, the degree of translation between representations can be varied. DEMIST is designed to support five levels of translation, although it currently offers these three levels of translation:

- *independent*, i.e. actions on this representation are not reflected onto other representations

- *map relation*, i.e. a learner can select a value in one representation and find all the corresponding relationships in the other representations (e.g. select a point on a X-Time graph and be shown the appropriate row in a table, or place on a histogram)
- *dyna-linked*, if a learner modifies the information in one representation, this action is reflected onto all the other relevant representations.

There are a small number of additional activities available to the learners. In particular, they can make *hypotheses* about the values of the model in the future or perform *actions*, which allows the learner to act on a value at the current stage of the simulation and change it. They can choose which representations they use to perform these activities and depending on the degree of translation could check the consequences of these actions on other representations (e.g. predict that the population density will have doubled in size in 10 years by adding a hypothesis to the relevant row of a table and see a point added to the graph corresponding to that prediction).

Study 1

We are currently conducing studies to evaluate if DEMIST is effective at supporting understanding on the concepts and representations of population dynamics. We also are exploring the decisions learners make when provided with many complex representations. In these studies DEMIST was authored to provide three scenarios that contained a number of learning units. Table 1 shows how these were designed with respect to the DeFT parameters.

Table 1. Design Parameters of DEMIST in Study 1

Design Issue	Decision
Information	Representations created with one, two or three dimensions of information. Pairs of representations could therefore have full, partial or no redundancy
Form	Large representational system (between 8 and 10 ERs for each units), which varied in their relevance and ease of interpretation.
Sequence	Learner choice of sequence of representations and when to swap
Number	A maximum of five co-present representations. A small number of representations were selected to be displayed at the beginning of each unit.
Translation	Dyna-linking allowing learners to reflect actions onto other representations

Pre and post-test material was multiple choice and included conceptual questions, (e.g. what will happen to the prey population if some predators are removed?), specific representations items (e.g. which of these graphs of population density against time is characteristic of a single species growing in an unlimited manner?) and multi-representational items (e.g. finding the odd-one-out among four different representations of supposedly the same dataset). These questions were designed to assess if multi-representational simulations such as DEMIST can support learning about representations and the relation between representations as well as the more traditional conceptual issues.

Results

A pre-test revealed that the 18 undergraduates in this study had limited prior knowledge of the domain. Chance performance is 25% as each question had one right answer and three distractors. The average pre-test score was 42.3%, which is significantly above chance ($t= 4.3$, $df = 16$, $p < 0.001$). Closer analysis revealed questions of different types were answered differently. At pre-test, Conceptual and ER question were answered above chance; those relating to MERs were answered below chance. The post-test consisted of 10 items from the pre-test and 12 more items. Again the performance of participants was significantly above chance at 55.6%, ($t= 8.5$, $df = 16$, $p<.0001$). Table 1 shows that overall there was a significant increase in the percentage of questions that subjects got right from pre-test to post-test ($t = 3.1$, $df =16$, $p<0.008$). As the post-test included more difficult items than the pre-test, we compared subjects' performance on those questions that were present on both the pre and post-test. Scores significantly improved on these questions from an average of 45.9% at pre-test to 62.3% at post-test ($t = 4.9$, $df = 16$, $p <.0001$). Finally, we looked at performance on post-test items by type of question. Performance on all questions was now significantly above chance except for those questions that dealt specifically with MERs.

Table 2. Pre-Test and Post-Test results

	Overall		*Concept*		*Single ER*		*MERs*	
	Mean	*St.Dev.*	*Mean*	*St.Dev.*	*Mean*	*St.Dev.*	*Mean*	*St.Dev.*
Pre-Test (11 items)	42.3%	16%	41.2%	23%	52.9%	21%	11.8%	22%
Post-Test (22 items)	55.6%	15%	61.3%	27%	59.9%	10%	33.8%	26%

The second goal of the study was to explore learners' representation use to discover whether they had strong preferences about the representations. Learners had the choice to work simultaneously with between one and five representations plus the controller. The majority of learners spend most of their time working with three representations (40% of total time) or four representations (32%). Participants tended to explore as much as possible of the representational space, activating an average of 73 representations out of the 80 available. However, this does imply that they used the representations equally. To examine which representations learners preferred we calculate the amount of time each type of representation was used. We found that the following preferences for representations (order by percentage of time used as a function of availability): X v Time Graph> Terms > Value > Chart > XY Graph > Animation > Table > Equation > Pie Chart > X v Time Graph (log). The first analysis performed on this data examined how influenced learners had been by the initial selection of representations for a unit. For each unit we had chosen two or three representations to open automatically. However, learners had been free to close those representations at any time. We found a striking correlation between our provision of representations and the ones that learners spent the most time working with ($r = 0.85$, $n = 10$, $p < 0.02$). Thus, a large degree in the variance of use is not based on a learner's choice of representations; it is based on the system's choice. The ERs that learners selected for different amounts of time than that predicted simply by automatic selection include the XY graph which was used more than expected, and the table and animation, which were used less. The trace logs provided further information about which ER was associated with initiating actions as well in addition to their display function. This was examined for translation requests and for hypotheses. There is enormous variance in how the representations were used for action. The X v Time Graph was used for 98% of all hypotheses. For translation requests, again the X v Time Graph was the most common accounting for 58% of all requests, but the XY Graph (25%) and the Table (12%) were also used appreciably. These latter figures are particularly interesting, as they do not reflect the percentage of time that learners chose to display the representations. Translation requests from the XY graph are plausibly about trying to understanding a new and difficult to interpret representation, whereas from the table perhaps its familiarity was being used by learners to help interpret other representations.

Conclusion

This paper has presented the DeFT framework for learning with multiple representations and a simulation environment which has been designed to implement it. It also reports on an initial study that has revealed DEMIST to be a suitable environment to ask questions about how learners should best be supported when they learn with MERs. It is based on a rich domain, which is best understood by reference to multiple linked representations. We have shown that students can begin to understand the domain in a short amount of time but that the more complex issues will require more time and strategic support. By analysing learners' representation use we have found that although they use many of DEMIST's functions but that they rely heavily on the author's decisions about representations to display. Perhaps because of this, no systematic relationships have been found which relate learners' use of representations to either prior knowledge of learning outcomes. A second study is now underway which has no initial ERs displayed for learners. This should provide further insight into how much guidance a supposedly discovery environment like DEMIST ought to provide and may reveal more information how learners' preferences for representations impacts upon their learning. Using detailed protocol analysis we hope to build a more complete picture of the process of learning in this domain and ultimately this should form the basis of a computational model (e.g. Tabachneck-Schijf, et al, 1997). Second, we can perform design experiments, which systematically vary the DeFT parameters (e.g. amount of translation, number of co-present representations). A combination of these two approaches should help uncover further design principles for how best to support the complex information processing that MERs require.

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